

# Workshop on Set Optimization and Dynamic Systems

## Abstracts

### **E. Rosazza-Gianin, New perspective to capital allocation rules**

*Based on joint works with F. Centrone and E. Mastrogiacomo.*

In the theory of risk measures, capital allocation is a well-known problem consisting in sharing “ad hoc” the margin required for a position among the different sources of riskiness. Such a problem has been faced in the literature by using different approaches (depending on the nature of the risk measure behind) also in connection with systemic risk and game theory. Although there is a wide literature on the relation between dynamic risk measures and BSDEs and on capital allocation rules in a static setting, only a few recent papers on capital allocation work in a dynamic setting by focusing mainly on the gradient approach. In this talk, we discuss new perspectives to the capital allocation problem going beyond those already existing in the literature. In particular, a set-valued approach arises naturally once the case of nondifferentiability is covered for capital allocation rules based on subdifferentials. Furthermore, a general approach to capital allocations in a dynamic framework for risk measures induced by  $g$ -expectations will be introduced and studied in an axiomatic way.

### **D. Visetti, Does a multiobjective optimization problem lead to Euler-Lagrange equations? A set-valued approach**

*Joint work with F. Heyde.*

We provide some answers to the problem of minimizing an integral functional of a vector-valued Lagrangian on a set of admissible arcs with given endpoints. The vector-valued problem is embedded into a set-optimization problem, so that a complete lattice approach is considered. It is so possible to give a definition of a minimizer and of an infimizer, as two completely different concepts.

Set-valued Euler-Lagrange equations are obtained. Following the guidelines of the classical results, under convex and coercive hypotheses an existence result of an infimizer is proved. Moreover, the constrained problem is studied in the

isoperimetric case. An application is given to the optimization of the shape of energy-saving buildings.

This project was funded by VerTecMVP, Free University of Bozen.

## **F. Bagagiolo, Hysteresis and some vector/set-valued problems**

Hysteresis is a kind of memory effect between a time-dependent input and a time-dependent output. Its special feature is its rate-independence, that is: the history of the output only depends on the sequence of values reached by the input along its history and not on how much fast the input runs its history. The rate-independence makes hysteresis suitable in order to perform a possible scalarization of some optimal control problems with a vectorial objective function. I will give some examples of that. Moreover, the mathematical representation of the hysteresis effects is often linked to some particular movement of sets in  $\mathbb{R}^n$  (as the so-called sweeping process). Still using the rate-independence, I will give some results about the controllability of such a process.

## **Z. Feinstein, Dynamic Programming Principle for Nonzero Sum Games**

*This is joint work with Birgit Rudloff and Jianfeng Zhang.*

Nonzero sum games typically have multiple Nash equilibriums (or no equilibrium), and unlike the zero sum case, they may have different values at different equilibriums. In this talk, instead of focusing on the existence of individual equilibriums, we study the set of values over all equilibriums, which we call the set value of the game. The set value is unique by nature and always exists (with possible empty value). Similar to the standard value function in control literature, it enjoys many nice properties such as regularity, stability, and more importantly the dynamic programming principle. We shall consider both discrete and continuous time models with finite time horizon.

## **A.H. Hamel/F. Heyde, The translativity property in finance as a dynamic feature - primal and dual interpretation: an impulse for discussion**

Cash-additivity of risk measures is a well-studied property in financial math. It is often overlooked that it comes from a dynamic feature, namely a discounting procedure. A more general version of it is proposed in this short contribution, and its direct=primal as well as its dual interpretation is discussed. Along the way, conditional expectations will appear as adjoints of embedding operators.